USE OF AUXILIARY INFORMATION IN TWO STAGE SUCCESSIVE SAMPLING

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1. INTRODUCTION

In successive sampling information collected on the matched portion of the sample is used to improve the estimate of the population character under study. The theory has been developed by Jessen (1942), Yates (1949), Patterson (1950), Tikkiwal (1951, 55, 56), Eckler (1955) etc. The theory of successive sampling in the case of multi-stage design has been considered by Tikkiwal (964, 65), Singh, D. (1968), Singh, D. and Kathuria, O.P. (1969), Abraham et al (1969) and Singh, S. (1970) etc. When auxiliary information on some characters highly correlated with the character under study is also available it can further be utilised to improve the estimate. Estimates of population mean utilising auxiliary characters have been discussed in one stage successive sampling by J.A. Sastri (1970). In this paper an attempt has been made to provide minimum variance unbiased linear estimates of (*i*) the population mean on the most recent occasion (*ii*) the change in the population mean from one occasion to another and (*iii*) an overall estimate of population mean overall occassions, for a two stage design in which primary stage units are partially replaced and the second stage units in the retained primaries have been kept fixed. The entire study has been made under general correlation model.

2. A RESULT

For the sake of clarity and completeness it is worthwhile to consider a well known result regarding minimisation of the variance of a linear function. Let there be a function of type

$$f(\underset{\sim}{a})=\underset{\sim}{a}'\underset{\sim}{x}+y \qquad \text{...(2.1)}$$

where $\underset{\sim}{a}$ and $\underset{\sim}{x}$ are column vectors of coefficients and random variables respectively. If variance covariance matrix of $\underset{\sim}{x}$ is Σ_0 and covariance of y with $\underline{x}$ is $-\underline{z}$ such that cov $(y, x_i) = -z_i$, then varniace of $f(\underline{a})$ is given by

$$V f(\underline{a}) = \underline{a}' \Sigma_0 \underline{a} + V(y) - 2\underline{a}' \underline{z}$$

Minimising this variance with respect to a's, it followo that $\underline{a}$ can be obtained by solving the equations

$$\Sigma_0 \underline{a} = \underline{z} \qquad \ldots(2.2)$$

as

$$\underline{a} = \Sigma_0^{-1} \underline{z} \qquad \ldots(2.3)$$

and

$$Min\ V f(\underline{a}) = V(y) - \underline{z}' \Sigma_0^{-1} \underline{z} \qquad \ldots(2.4)$$

3. Estimation of mean

3.1 Sampling on h occasions:

Assume that the population considered consists of 'N' primary stage units (psu's), each containing 'M' second stage units (ssu's).

Following the simple random sampling at both the stages on the first occasion select 'n' psu's and from each selected psu select 'm' ssu's. Selection at both stages being without replacement. A sub-sample of size 'np' of the selected psu's along with their ssu's is retained on the second occasion and is supplemented by selecting afresh 'nq' psu's from the units not selected on the first occasion, $(p+q=1)$. In each of 'nq' psu's selection of ssu's is done as in the case of first occasion. The psu's retained during the second occasion have been retained alongwith their ssu's in all the subsequent occasions whereas the remaining 'nq' psu's have been selected afresh at each occasion from the units not selected on previous occasion. We assume further that (*i*) the population considered is large (*ii*) the sample size is constant on each occasion and (*iii*) any commonness of units between two occasions in 'nq' units selected afresh every time is ignored. When the information on K auxiliary characters $X_1, X_2, \ldots X_k$ which are highly correlated with the character under study Y is available, an unbiased linear estimate of the population mean for the hth occasion can be put as,

$$\bar{y}_h = \sum_{l=1}^{k} \sum_{v=1}^{h+1} a_{lv}(\bar{x}_{lv} - \bar{X}_l) + \sum_{t=1}^{h} b_t(\bar{y}_t' - \bar{y}_t'') + \bar{y}_h'' \qquad \ldots(3.1.1)$$

where

$\overline{X}_l$: Population mean of the l^{th} auxiliary character

$\overline{Y}_t$: Population mean of the character under study at the t^{th} occasion.

$\bar{x}_{lv}$: mean per ssu of the l^{th} auxiliary variate based on

(*i*) nmp units which are common to all occasions for $v=1$ and all l;

(*ii*) and nmq units which are selected afresh for $v=2$ to $h+1$ and all l,

$\bar{y}'_t$: mean per ssu of the variate under study at the t^{th} occasion for the nmp units which are common to all occasions.

$\bar{y}''_t$: mean per ssu of the variate under study at the t^{th} accasion for nmq units which are selected afresh in the t^{th} occasion.

We define.

$$\alpha^*_l = S^2_{bx_l} + \frac{S^2_{wx_l}}{m}$$

$$\alpha_t = S^2_{by_t} + \frac{S^2_{wy_t}}{m}$$

$$\beta^{**}_{ll'} = \rho'^{**}_{ll'} S_{bx_l} S_{bx_{l'}} + \rho''^{**}_{ll'} \frac{S_{wx_l} S_{wx_{l'}}}{m}$$

$$\beta^*_{lt} = \rho^{*\prime}_{lt} S_{bx_l} S_{by_t} + \rho''^*_{lt} \frac{S_{wx_l} S_{wv_t}}{m}$$

$$\beta_{tt'} = \rho'_{tt'} S_{by_t} S_{by_{t'}} + \rho''_{tt'} \frac{S_{wy_t} S_{wy_{t'}}}{m}$$

where,

$$S^2_{bx_l} = \frac{1}{N-1} \sum_{i=1}^{N} (\overline{X}_{li} - \overline{X}_l)^2$$

$$S^2_{wx_l} = \frac{1}{N(M-1)} \sum_{i=1}^{N} \sum_{j=1}^{M} (X_{lij} - \overline{X}l_i)^2$$

$$S^2_{by_t} = \frac{1}{N-1} \sum_{i=1}^{N} (\bar{Y}_{ti} - \bar{Y}_t)^2$$

$$S^2_{wy_t} = \frac{1}{N(M-1)} \sum_{i=1}^{N} \sum_{j=1}^{M} (Y_{tij} - \bar{Y}_{ti})^2$$

$$\rho'^{**}_{ll'} \, S_{bx_l} \, S_{bx_{l'}} = \frac{1}{N-1} \sum_{i=1}^{N} (\bar{X}_{li} - \bar{X}_l) \, (\bar{X}'_{l'i} - \bar{X}'_{l'})$$

$$\rho''^{**}_{ll'} \quad S_{wx_l} \, S_{wx_{l'}} = \frac{1}{N(M-1)} \sum_{i=1}^{N} \sum_{j=1}^{M} (X_{lij} - \bar{X}_{li}) \, (X_{l'ij} - \bar{X}_{l'i})$$

$$\rho'^{*}_{lt} \, S_{bx_l} S_{by_t} = \frac{1}{N-1} \sum_{i=1}^{N} (\bar{X}_{li} - \bar{X}_l) \, (\bar{Y}_{ti} - \bar{Y}_t)$$

$$\rho''^{*}_{lt} \quad S_{wx_l} S_{wy_t} = \frac{1}{N(M-1)} \sum_{i=1}^{N} \sum_{j=1}^{M} (X_{lij} - \bar{X}_{li})(\bar{Y}_{tij} - \bar{Y}_{ti})$$

$$\rho'_{tt'} \, S_{by_t} S_{by_{t'}} = \frac{1}{N-1} \sum_{i=1}^{N} (\bar{Y}_{ti} - \bar{Y}_t) \, (\bar{Y}'_{t'i} - \bar{Y}'_{t'})$$

$$\rho''_{tt'} \, S_{wy_t} S_{wy_{t'}} = \frac{1}{N(M-1)} \sum_{i=1}^{N} \sum_{j=1}^{M} (Y_{tij} - \bar{Y}_{ti})(Y'_{t'ij} - \bar{Y}'_{t'i})$$

$\rho'^{**}_{ll'}$ = correlation between the psu means of the l^{th} and l'^{th} auxiliary variates. ($l \neq l'$ and $l, l' = 1, 2, \ldots k$)

$\rho''^{**}_{ll'}$ = correlation between the ssu means of the l^{th} and l'^{th} auxiliary variates. ($l \neq l'$ and $l, l' = 1, 2, \ldots k$)

ρ'^{*}_{lt} = correlation between the psu means of the l^{th} auxliary variate and the variate under study at the t^{th} occasion.

ρ''^{*}_{lt} = correlation between the ssu means of the l^{th} auxiliary variate and the variate under study at the t^{th} occasion.

$\rho'_{tt'}$ = correlation between the psu means of the variate under study at the t^{th} and t'^{th} occasion.

$\rho''_{tt'}$ = correlation between the ssu means of the variate under study at the t^{th} and t'^{th} occasion.

Y_{tij} = the observation of the character under study on the j^{th} ssu, in the i^{th} psu at the t^{th} occasion.

$$\bar{Y}_{ti} = \frac{1}{M} \sum_{j=1}^{M} Y_{tij}$$

X_{lij} = the observation of the l^{th} auxiliary variate on the j^{th} ssu, in the i^{th} psu.

$$\bar{X}_{li} = \frac{1}{M} \sum_{j=1}^{M} X_{lij}$$

Optimum values of a_{lv}'s ($l=1, 2, \ldots k$; $v= 1, 2, \ldots h+1$) and b_t's ($t=1, 2, \ldots h$) which will minimise the $V(\bar{y}_h)$ can be obtained from the equation

$$XA=B \qquad \ldots\ldots\ldots \qquad (3.1.2.)$$

where X is a matrix of dimensions $\{k(h+1)+h,\ k(h+1)+h\}$.

$$X = \begin{pmatrix} X_{11} & X_{12} \ldots\ldots\ldots X_{1k} & C_1 \\ X_{21} & X_{22} \ldots\ldots\ldots X_{2k} & C_2 \\ \ldots\ldots & \ldots\ldots\ldots\ldots\ldots\ldots\ldots & \ldots \\ \ldots\ldots & \ldots\ldots\ldots\ldots\ldots\ldots\ldots & \ldots \\ X_{k1} & X_{k2} \ldots\ldots\ldots X_{kk} & C_k \\ C'_1 & C'_2 \ldots\ldots C'_k & D \end{pmatrix}$$

where

$$X_{ll'} = \left(\left(\eta_{tt'}^{ll'}\right)\right) \qquad t, t'=1, 2, \ldots h+1 \quad l, l'=1, 2, \ldots k$$

$$\eta^{ll'}_{tt} = \alpha^*_l \qquad \text{when } l=l'$$

$$= \beta^{**}_{ll'} \qquad \text{when } l \neq l'$$

$$\eta^{ll'}_{tt'} = 0 \qquad \text{when } t \neq t'$$

$$C_l = \left(\begin{pmatrix} E_l \\ F_l \end{pmatrix}\right) \qquad l=1, 2, \ldots k$$

$E_l = (\beta^*_{l1}, \beta^*_{l2}, \ldots \beta^*_{lh})$ is a row vector of h elements

$$F_l = ((\gamma^l_{tt'})) \qquad t, t'=1, 2, \ldots h$$

$$\gamma^l_{tt'} = -\beta^*_{lt} \qquad \text{when } t=t'$$

$$= 0 \qquad \text{when } t \neq t'$$

$$D = ((\xi_{tt'})) \qquad \text{when } t, t'=1, 2, \ldots h$$

$$\xi_{tt'} = \alpha_t \qquad \text{when } t=t'$$

$$= q\beta_{tt'} \qquad \text{when } t \neq t'$$

$$A = [a_{11}, a_{12} \ldots a_{1,h+1} \,;\, a_{21}, a_{22}, \ldots a_{2,h+1};\, a_{k1}, a_{k2}, \ldots a_{k,h+1} \,;\, b_1, b_2 \ldots b_h]$$

and

$$B = [0, 0, \ldots, -\beta^*_{1h} \,;\, 0, 0, \ldots, -\beta^*_{2h} \,;\, \ldots;\, 0, 0, \ldots -\beta^*_{kh} \,;\, 0, 0 \,.\, p\alpha_h]$$

A and B are column vectors of $(k(h+1)+h)$ elements.

It can be seen that the vectors A and B correspond to vectors a and z of section 2 and the matrix x and $\bar{y}''_h$ correspond to Σ_o and y therein. The variance in (2.4) in this case, after a little simplification reduces to

$$V(\bar{y}_h) = \frac{1}{nq}\left(1 - b_h\right)\frac{P_h}{P_o} \qquad \ldots(3.1.3)$$

where

$$P_t = \begin{vmatrix} \alpha^*_1 & \beta^{**}_{12} & \beta^{**}_{13} & \cdots & \beta^{**}_{1k} & \beta^*_{1t} \\ \beta^{**}_{12} & \alpha^*_2 & \beta^{**}_{23} & \cdots & \beta^{**}_{2k} & \beta^*_{2t} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \beta^{**}_{1k} & \beta^{**}_{2k} & \cdots & \cdots & \alpha^*_k & \beta^*_{kt} \\ \beta^*_{1t} & \beta^*_{2t} & \cdots & \cdots & \beta^*_{kt} & \alpha_t \end{vmatrix}$$

is a determinant and P_o is the determinant obtained by omitting the last row and last column of P_t.

Particular Cases

3.2. *Sampling on h occasions without using auxiliary characters*

The estimate of mean at the h^{th} occasion and its variance can be deduced from equations (3.1.1) and (3.1.3) respectively as

$$(\bar{y}_h)_o = \sum_{t=1}^{h} b_t(\bar{y}'_t - \bar{y}''_t) + \bar{y}''_h \qquad \ldots(3.2.1)$$

and

$$V(\bar{y}_h)_o = \frac{\alpha_h}{nq}\left(1 - b_h\right) \qquad \ldots(3.2.2)$$

where

$$b_t = p\alpha_h \frac{\triangle_{ht}}{\triangle_h} \qquad t = 1, 2, \ldots h$$

and $\triangle_h$ is the determinant of the matrix D, and $\triangle_{ht}$ is the cofactor of the element common to the h^{th} row and t^{th} column in $\triangle_h$. This particular case has been discussed in reference (10).

3.3 *Sampling on two occasions with one auxiliary character*

Substituting $h=2$ and $k=1$ in (3.1.1), we get

$$\bar{y}_2 = \sum_{v=1}^{3} a_{1v}(\bar{x}_{1v} - \bar{X}_1) + \sum_{t=1}^{2} bt(\bar{y}'_t - \bar{y}''_t) + \bar{y}_2'' \quad \ldots(3.3.1$$

where

$$a_{11} = -\frac{pt_2}{\alpha^*_1 \triangle}\left[t_1\beta^*_{12} - qt_2\beta^*_{11}\right]$$

$$a_{12} = -\frac{\beta^*_{11}}{\alpha^*_1 \triangle}\left[pqt_2t_3\right]$$

$$a_{13} = -\frac{q\beta^*_{12}}{\alpha^*_1 \triangle}\left[t_1t_2 - qt^2_3\right]$$

$$b_1 = -\frac{pqt_2t_3}{\triangle}$$

$$b_2 = \frac{pt_1t_2}{\triangle}$$

$$t_1 = \alpha_1 - \frac{\beta^{*2}_{11}}{\alpha^*_1}$$

$$t_2 = \alpha_2 - \frac{\beta^{*2}_{12}}{\alpha^*_1}$$

$$t_3 = \beta_{12} - \frac{\beta^*_{11}\beta^*_{12}}{\alpha^*_1}$$

and

$$\Delta = t_1 t_2 - q^2 t_3^2.$$

Now variance $\bar{y}_2$ would be

$$V(\bar{y}_2) = \frac{\alpha_2}{n}\left(1 - \frac{\beta^{*2}_{12}}{\alpha^*_1}\right)\left[\frac{t_1 t_2 - q t^2_3}{t_1 t_2 - q^2 t_3^2}\right] \qquad (3.3.2)$$

In this case, when no auxiliary information is employed $t_1 = \alpha_1$; $t_2 = \alpha_2$ and $t_3 = \beta_{12}$ then equations (3.3.1) and (3.3.2) become as,

$$(\bar{y}_2)_o = -\frac{pq\beta_{12}\alpha_2}{\alpha_1\alpha_2 - q^2\beta_{12}^2}\left(\bar{y}_1' - \bar{y}_1''\right) + \frac{p\alpha_1\alpha_2}{\alpha_1\alpha_2 - q^2\beta_{12}^2}\left(\bar{y}_2' - \bar{y}_2''\right) + \bar{y}_2'' \qquad \ldots(3.3.3)$$

and

$$V(\bar{y}_2)_o = \frac{\alpha_2}{n}\left(\frac{\alpha_1\alpha_2 - q\beta^2_{12}}{\alpha_1\alpha_2 - q^2\beta^2_{12}}\right) \qquad \ldots(3.3.4)$$

3.4. *Unistage design*

It is easy to see that when the design is unistage, putting $m = M$ we have

$$\alpha_t = S^2_{by_t}$$

$$\alpha^*_l = S^2_{bx_l}$$

$$\beta_{tt'} = \rho'_{tt'}\ S_{by_t}\ S_{by_{t'}}$$

$$\beta^*_{lt} = \rho'^*_{lt}\ S_{bx_l}\ S_{by_t}$$

and

$$\beta^{**}_{ll'} = \rho'^{**}_{ll'}\ S_{bx_l}\ S_{bx_{l'}}$$

Now with these definitions of α_t, α^*_l, $\beta_{tt'}$, β^*_{lt} and $\beta^{**}_{ll'}$ we deduce the estimate and variance from (3.1.1) and (3.1.3) respectively. In particular, if there are two occasions and one auxiliary character then

$$V(\bar{y}_2) = \frac{S^2_{by_2}}{n}\left(1 - \rho'^*_{12}\right)\left\{\frac{(1-\rho'^*_{11})(1-\rho'^*_{12}) - q(\rho'_{12} - \rho'^*_{11}\,\rho'^*_{12})^2}{(1-\rho'^*_{11})(1-\rho'^*_{12}) - q^2(\rho'_{12} - \rho'^*_{11}\,\rho'^*_{12})^2}\right\} \qquad \ldots(3.4.1)$$

This case has been discussed in reference (7).

4. ESTIMATE OF CHANGE BETWEEN $(h-1)^{th}$ AND h^{th} OCCASION

4.1 *An estimate utilising k auxiliary characters*

In the course of time, when information is available on a number of occasions, we can improve the estimates on earlier occasions utilising the information collected on subsequent occasions. An unbiased linear estimate of change between the current estimate at the h^{th} occasion and an improved estimate at the $(h-1)^{th}$ occasion can be obtained as,

$$C_{h,h-1}=\sum_{l=1}^{k}\sum_{v=1}^{h+1} a'_{lv}(\bar{x}_{lv}-\bar{X}_l)+\sum_{t=1}^{h} b_t'(\bar{y}_t'-\bar{y}_t'')+\bar{y}_h''-\bar{y}''_{h-1} \quad ...(4.1.1)$$

and minimum variance as

$$V(C_{h,\,h-1})=\frac{1}{nq}\left(1+b'_{h-1}\right)\frac{P_{h-1}}{P_o}+\frac{1}{nq}\left(1-b'_h\right)\frac{P_h}{P_o} \quad ...(4.1.2)$$

In this case a'_{lv}'s$(l=1, 2,...k\,;\ v=1, 2,...h+1)$ and b'_t's $(t=1, 2,...h)$ are obtained from the equation given by

$$XA_o=B_o \quad ...(4.1.3)$$

where

$$A_o=[a'_{11}, a'_{12}, ..a'_{1,h+1}\,; a'_{21}, a'_{22},...a'_{2,h+1}\,;\,...\,;\, a'_{k1}, a'_{k2},...a'_{k,h+1}\,; b'_1, b'_2...b'_h]$$

and

$$B_o=[0, 0, 0,...,-\beta^*_{1h}\,;\, 0,0,0,...-\beta^*_{2h}\,;......;\, 0,0,0,...-\beta^*_{kh}\,;\, 0,0,0,...-p\alpha_{h-1}, p\alpha_h]$$

A_o and B_o are column vectors of $\{K(h+1)+h\}$ elements. It is seen that the vectors A_o and B_o correspond to $\underline{a}$ and $\underline{z}$ of section 2. and $(\bar{y}''_h-\bar{y}''_{h-1})$ stands for y therein.

Particular Cases

4.2. *Sampling on h occasions without using auxiliary characters*

When no auxiliary information is employed an estimate of change between h^{th} and $(h-1)^{th}$ occasion is given by

$$(C_{h,h-1})_o=\sum_{t=1}^{h} b'_t(\bar{y}'_t-\bar{y}''_t)+\bar{y}''_h-\bar{y}''_{h-1} \quad ...(4.2.1)$$

and variance of $(C_{h,h-1})_o$ is

$$V(C_{h,h-1})_o=\frac{\alpha_{h-1}}{nq}\left(1+b'_{h-1}\right)+\frac{\alpha_h}{nq}\left(1-b'_h\right) \quad ...(4.2.2)$$

where b'_t's are obtained as

$$b'_t = \frac{1}{\triangle_h}\left[p\alpha_h \triangle_{ht} - p\alpha_{h-1}\triangle_{h-1,\ t}\right]$$

4.3. *Sampling on two occasions with one auxiliary character*

In this particular case putting $h=2$ and $k=1$ in (4.1.1) it is seen that

$$C_{12} = \sum_{v=1}^{3} a'_{1v}(\bar{x}'_{1v} - \bar{X}_1) + \sum_{t=1}^{2} b'_t(\bar{y}'_t - \bar{y}''_t) + \bar{y}_2'' - \bar{y}_1'' \quad \ldots(4.3.1)$$

where

$$a'_{11} = \frac{-p}{\alpha^*_1\triangle}\left[\beta^*_{12}(t_1t_2 + qt_1t_3) - \beta^*_{11}(t_1t_2 + qt_2t_3)\right]$$

$$a'_{12} = \frac{q\beta^*_{11}}{\alpha^*_1\triangle}\left[t_1t_2 - qt_3^2 - pt_2t_3\right]$$

$$a'_{13} = \frac{-q\beta^*_{12}}{\alpha^*_1\triangle}\left[t_1t_2 - qt_3^2 - pt_1t_3\right]$$

$$b'_1 = \frac{-pt_2}{\triangle}\left(t_1 + qt_3\right)$$

and

$$b'_2 = \frac{pt_1}{\triangle}\left(t_2 + qt_3\right)$$

Now

$$V(c_{12}) = \frac{1}{n\triangle}\left[(t_1 + t_2)(t_1t_2 - qt_3^2) - 2pt_1t_2t_3\right] \quad \ldots(4.3.2)$$

when no auxiliary information is available, then (4.3.2) would be,

$$V(c_{12})_o = \frac{1}{n}\left\{\frac{(\alpha_1 + \alpha_2)(\alpha_1\alpha_2 - q\beta_{12}^2) - 2p\alpha_1\alpha_2\beta_{12}}{\alpha_1\alpha_2 - q^2\beta_{12}^2}\right\} \quad \ldots(4.3.3)$$

5. Estimate of overall mean

5.1. *Sampling on h occasions with k auxiliary characters*

An overall unbiased linear estimate of the population mean over h occasions for the type of sampling pattern considerd can be

$$E_h = \sum_{l=1}^{k} \sum_{\nu=1}^{h+1} a''_{l\nu}(\bar{x}_{l\nu} - \bar{X}_l) + \sum_{t=1}^{h} \theta_t \{b_t''(\bar{y}'_t - \bar{y}''_t) + \bar{y}''_t\} \qquad (5.1.1)$$

where θt's $(t=1, 2...h)$ are some suitable weights depending upon the relative importance of the occasions, such that $\sum_{t=1}^{h} \theta_t = 1$. Optimum values of $a''_{l\nu}$'s $(l=1, 2,...k\,;\ \nu=1, 2,...h+1)$ and $b_t'''s$ $(t=1, 2,...h)$ which will minimise the variance $V(E_h)$ are obtained from the equations

$$YA_1 = B_1$$

where Y is a matrix of dimension $\{k(h+1)+h,\ k(h+1)+h\}$

$$Y = \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1k} & G_1 \\ X_{21} & X_{22} & \cdots & X_{2k} & G_2 \\ \cdots & \cdots & & \cdots & \cdots \\ \cdots & \cdots & & \cdots & \cdots \\ X_{k1} & X_{k2} & \cdots & X_{kk} & G_k \\ G_1' & G_2' & \cdots & G_k' & D_1 \end{bmatrix}$$

where X_{ll}' is as defined in Section 3.

$$G_l = \left(\begin{pmatrix} Q_l \\ R_l \end{pmatrix}\right) \qquad l=1, 2, \ldots\ldots, k$$

$Q_l = (\theta_1\beta^*_{l1}, \theta_2\beta^*_{l2}, \ldots\ldots, \theta_h\beta^*_{lh})$ is a row vector of h elements

$R_l = ((\delta^l_{tt'}))$ $\qquad t, t'=1, 2...h$

$\delta^l_{tt'} = -\theta_t\beta_{lt}$ $\qquad$ when $t=t'$

$\quad = 0$ $\qquad$ when $t \neq t'$

$D_1 = ((\psi_{tt'}))$ $\qquad t, t'=1, 2.. h$

$\psi_{tt'} = \theta_t\alpha_t$ $\qquad$ when $t=t'$

$\quad = \theta_t' q\beta_{tt'}$ $\qquad$ when $t \neq t'$

$$A_1 = [a''_{11}, a''_{12}, \ldots a''_{1,h+1}\,;\ a''_{21}, a''_{22}, \ldots a''_{2,h+1}\,;\ \ldots\,;\ a''_{k1}, a''_{k2}, \ldots a''_{k,h+1}\,;\ b''_1, b''_2, \ldots b''_h]$$

and

$$B_1=[0, -\theta_1\beta^*_{11}, -\theta_2\beta^*_{12}, \ldots -\theta_h\beta^*_{1h}; \ldots; 0, -\theta_1\beta^*_{k1}, -\theta_2\beta^*_{k2}, \ldots -\theta_h\beta^*_{kh}; p\theta_1\alpha_1, p\theta_2\alpha_2 \ldots p\theta_h\alpha_h]$$

A_1 and B_1 are column vectors of $\{k(h+1)+h\}$ elements.

Now minimum variance of the estimate E_h would be,

$$V(E_h)=\frac{\theta_1^2}{nq}\left(1-b_1''\right)\frac{P_1}{P_o}+\frac{\theta_2^2}{nq}\left(1-b_2''\right)\frac{P_2}{P_o}+\ldots+\frac{\theta_h^2}{nq}\left(1-b_h''\right)\frac{P_h}{P_o}$$

$$=\frac{1}{nq}\sum_{t=1}^{h}\theta_t^2(1-b_t'')\frac{P_t}{P_o} \quad \ldots (5.1.2)$$

Particular Cases

5.2. *Sampling on h occasions without using auxiliary characters*

When no auxiliary information is employed, an overall linear unbiased estimate of the population mean would be given by

$$(E_h)_o=\sum_{t=1}^{h}\theta_t\{b_t''(\bar{y}'_t-\bar{y}''_t)+\bar{y}''_t\} \quad \ldots (5.2.1)$$

and variance of $(E_h)_o$ would be

$$V(E_h)_o=\frac{1}{nq}\sum_{t=1}^{h}\theta_t^2(1-b_t'')\alpha_t \quad \ldots (5.2.2)$$

where

$$b_t''=\frac{p}{\theta_t\Delta_h}\sum_{l=1}^{h}\Delta_{lt}\,\alpha_l\theta_l.$$

5.3. *Sampling on two occasions with one auxiliary character*

Putting h 2 and $k=1$ in (5.1.1) it is seen that

$$E_2=\sum_{v=1}^{3}a''_{1v}(\bar{x}_{1v}-\bar{X}_1)+\sum_{t=1}^{2}\theta_t\{b_t''(\bar{y}'_t-\bar{y}''_t)+\bar{y}''_t\} \quad \ldots (5.3.1)$$

where

$$a''_{11}=-\left(\frac{\theta_1\beta^*_{11}}{\alpha^*_1}b_1''+\theta_2\frac{\beta^*_{12}b''_2}{\alpha^*_1}\right)$$

$$a''_{12} = -\frac{\theta_1 \beta^*_{11}}{\alpha_1^*}\left(1 - b_1''\right)$$

$$a_{13}'' = -\frac{\theta_2 \beta^*_{12}}{\alpha_1^*}\left(1 - b_2''\right)$$

$$b_1'' = \frac{pt_2}{\theta_1 \Delta}\left[\theta_1 t_1 - q\theta_2 t_3\right]$$

$$b''_2 = \frac{pt_1}{\theta_2 \Delta}\left[\theta_2 t_2 - q\theta_1 t_3\right].$$

Now variance E_2 would be

$$V(E_2) = \frac{1}{n\Delta}\left[(t_1 t_2 - qt^2_3)(\theta_1{}^2 t_1 + \theta_2{}^2 t_2) + 2pt_1 t_2 t_3 \theta_1 \theta_2\right] \qquad \text{...(5.3.2)}$$

when no auxiliary information is employed (5.3.2) becomes

$$V(E_2)_o = \frac{1}{n(\alpha_1\alpha_2 - q^2\beta_{12})}\left[(\alpha_1\alpha_2 - q\beta^2_{12})(\theta_1{}^2\alpha_1 + \theta_2{}^2\alpha_2) + 2p\alpha_1\alpha_2\theta_1\theta_2\beta_{12}\right] \qquad \text{...(5.3.3)}$$

If $\alpha_1 = \alpha_2 = \alpha$ (say) and $\beta_{12} = \beta$, then

$$V(E_2)_o = \frac{\alpha}{n}\left[\frac{(\theta_1{}^2 + \theta_2{}^2)(1 - q\beta/\alpha) + 2p\theta_1\theta_2\,\beta/\alpha}{1 - q^2\,\beta^2/\alpha^2}\right] \qquad \text{...(5.3.4)}$$

SUMMARY

In the present paper auxiliary characters have been utilised to build up unbiased linear estimates of:

(*i*) the population mean on the most recent occasion;

(*ii*) the change in the population mean from one occasion to another; and

(*iii*) the overall population mean for all the occasions in case of two-stage successive sampling.

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